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RADIATION FIELD OF AN ARBITRARY ANTENNA IN A MAGNETOPLASMA

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The problem of finding the far field of an antenna in a magnetoplasma such as the ionosphere has been solved by Bunkin[1] and a number of workers.[2,3,4,5,6] The object of this note is to rederive the result in a simpler manner and to express it in terms of the normalized characteristic plane waves of the medium. This has the advantage of showing clearly how the far field depends on the free space pattern of the antenna, on the shape of the dispersion surface, and on the polarization of the characteristic waves. In the derivation use is made of the magnetic field instead of the electric field. Because the polarization of this vector is easier to express, both the deduction of the result and its application to actual pattern computations are simplified.

Maxwell's equations for a lossless magnetoplasma, described by the Hermitian dielectric matrix K, are

$$\begin{aligned} \text{curl } E(r) &= -j\omega\mu_o H(r) - J_m(r) \\ \text{curl } H(r) &= j\omega\epsilon_o K\, E(r) + J_e(r) \end{aligned} \tag{1}$$

The source, given by the electric and magnetic current densities $J_e(r)$ and $J_m(r)$ can also be represented by a single vector

$$M(r) = J_m(r) - (1/j\omega\epsilon_o) \text{ curl } K^{-1} J_e(r). \tag{2}$$

Eliminating E in Equation (1) gives the equation

$$(\text{curl } K^{-1} \text{ curl } -k_o^2)\, H(r) = -j\omega\epsilon_o M(r) \tag{3}$$

that has to be solved for H. Taking the Fourier transform with respect to the space variable equation (3) becomes

$$\underline{G}(\underline{k})\ \underline{H}(k) = -j\omega\epsilon_o M(k) \tag{4}$$

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where k is the wave vector. From now on the variable will always be shown explicitly in functions of position while in the corresponding Fourier transform, denoted by the same letter, the wave-vector variable k will generally be omitted. Thus we will write $H(r)$ for the magnetic field but simply H, instead of $H(k)$, for its transform.

The matrix G may be written

$$G(k) = -k^2 \, U K^{-1} U - k_o^2. \tag{5}$$

Here the matrix U represents the operator $|k|^{-1} k \times$ which depends only on the direction of the vector k and not on its length. The corresponding geometric transformation is a projection on a plane perpendicular to k followed by a 90^o rotation about k.

The matrix $G(k)$ can be inverted by using the eigenvectors of the matrix $U K^{-1} U$. This matrix is Hermitian since, when the vector k is real, $U^{\dagger} = U^{T} = -U$. It has real eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and an orthonormal set of eigenvectors. The determinant of $U K^{-1} U$ being zero, only two eigenvalues λ_1 and λ_2 are different from zero. The corresponding eigenvectors H_1 and H_2 satisfy

$$-U K^{-1} U H_i = \lambda_i H_i \tag{6}$$

and can be normalized to make $|H_i|^2 = 1$, $(i = 1,2)$. Since (6) shows that H_1 and H_2 are perpendicular to k, we can complete the orthonormal set by taking for H_3 the unit vector u in the direction k. For symmetry we can write

$$-U K^{-1} U H_3 = \lambda_3 H_3$$

with $\lambda_3 = 0$.

The inverse of G becomes

$$G^{-1} = \sum_{i=1}^{i=3} \frac{H_i H_i^{\dagger}}{\lambda_i k^2 - k_o^2} \tag{7}$$

as can be verified by making use of (6). The solution for the magnetic field H(r) is obtained by the inverse Fourier transform of H(k) deduced from (4) and (7)

$$H(r) = -j\omega\epsilon_o(2\pi)^{-3} \iiint \Sigma \frac{H_i H_i^{\dagger} M}{\lambda_i k^2 - k_o^2} \; e^{-jk\cdot r} \; d^3k \tag{8}$$

Before finding the asymptotic value of H(r) when r is large let us consider the relation of the H_i and λ_i to the characteristic wave problem. To each direction in space, defined by a unit vector u, there correspond an operator U and therefore two eigenvalues λ_1, λ_2. The source free equation

$$G(k)\; H(k) = 0$$

has non-trivial solutions only when det G = 0, i.e., when $k_o^2/k^2 = \lambda_i$. Thus λ_1, λ_2 represent the inverse of the square of the refractive index in direction u for the ordinary and the extraordinary waves. The possible wave vectors in direction u are $\pm\, k_o \lambda_i^{-\frac{1}{2}}\, u$. The extremity of the wave vector describes a two-sheeted surface Σ, the dispersion surface, that has been considered by several authors.[4,9,10,11] To any point k on this surface corresponds a definite value of λ ($\lambda = k_o^2/k^2$) and a characteristic plane wave

$$F_k\;(r) = [E_k(0), H_k(0)]\; e^{-jk\cdot r} \tag{9}$$

The vector $H_k(0)$ can be taken as the H_i corresponding to $\lambda_i = \lambda$ and the vector $E_k(0)$ follows from Maxwell's equations

$$E_k(0) = -\frac{k}{\omega\epsilon_o}\; K^{-1} U\; H_k(0) \tag{10}$$

If the vector k makes the angle θ with the constant magnetic field H_0, the polarization ellipse of H_i has one axis in the plane (k, H_0) and the other perpendicular to it.[8] Furthermore, if the axial ratio is tan β, the angle β is simply related to θ and to the usual ionospheric parameters $X = \omega_N^2/\omega^2$ and $Y = \omega_H/\omega$, by

$$\tan 2\beta = 2 \cos\theta \csc^2\theta\, (1-X)Y^{-1}$$

Each of the two solutions of this equation for β correspond to one sheet of the surface Σ. The "ordinary" sheet Σ_o is usually defined as that which contains the wave vector $k_o \sqrt{1-X}$ for the direction perpendicular to H_0. For k on Σ_o the major axis of the H-ellipse is normal to the plane (k, H_0) and for k on the "extraordinary" sheet Σ_x the major axis is contained in the plane (k, H_0). This is illustrated in Figure 1.

In expressing the final result we shall normalize the field F_k to make the Poynting vector of length one. The Poynting vector (using rms values)

$$P = \mathrm{Re}\, E_k \times H_k^*$$

is normal to the dispersion surface at point k and its projection on k is simply related to $\left|H_k\right|^2$

$$u \cdot P = (\omega \mu_o / k) \left|H_k\right|^2$$

Therefore if a denotes the angle between P and H_0 (see Figure 1), the normalization requires that

$$\left|H_k\right|^2 = (k/\omega \mu_o) \cos (a - \theta) \tag{11}$$

instead of one.

Returning to the evaluation of the integral (8) for large values of r we shall proceed as in Lighthill[5]. Since the asymptotic contributions to the field come from the zeros of the denominator in the integrand, only the first two terms of (8) have to be considered. In a first step an integral such as

$$\iiint \frac{H_i H_i^{\dagger} M}{\lambda_i k^2 - k_o^2} \quad e^{-jk \cdot r} \, d^3k \tag{12}$$

is reduced to a surface integral over one sheet of Σ while the other integral is reduced to a surface integral over the other sheet. We can thus represent the sum of the two terms as a single integral of

$$\frac{H_k H_k^{\dagger} M}{\left|\nabla(\lambda k^2 - k_o^2)\right|} \qquad e^{-jk\cdot r}$$

over the whole of Σ (here λ is a single-valued function of k over Σ). More exactly, taking the radiation condition into account, the integral must be taken only over the portion Σ_+ of Σ where the Poynting vector P is directed toward the point of observation. As we have seen that P·k is positive this means that $(\alpha - \theta)$ must be an acute angle. The integral (12) becomes

$$2\pi j \iint_{\Sigma_+} \frac{H_k H_k^{\dagger} M}{\left|\nabla(\lambda k^2 - k_o^2)\right|} \quad e^{-jk\cdot r} d^2k \tag{13}$$

similar to equation (68) in Lighthill. The denominator is easily found since in the direction of k the derivative of $(\lambda k^2 - k_o^2)$ is simply $2\lambda k$. On Σ this reduces to $2k_o\sqrt{\lambda}$ and must also equal $\left|\nabla(\lambda k^2 - k_o^2)\right| \cos(\alpha - \theta)$ hence

$$\left|\nabla(\lambda k^2 - k_o^2)\right| = 2k_o\lambda^{1/2} \sec(\alpha - \theta)$$

This term reminds us of equation (11). If we change the normalization to make the Poynting vector of $F_k(r)$ equal to one we find indeed that the denominator disappears and that equation (8) reduces to

$$H(r) = \frac{(2\pi)^{-2}}{2} \iint_{\Sigma_+} H_k H_k^{\dagger} M e^{-jk\cdot r} d^2k \tag{14}$$

The next step is to reduce the surface integral to a sum over the stationary points for k·r. These are the points of tangency on Σ_+ of planes perpendicular to the direction of r. It has been observed by several authors[3,4,7] that there may exist up to 4 points having this property. It is sufficient to describe the term coming from one of these points as the same form applies to each of them. In the

vicinity of a stationary point the relevant property of the surface Σ is the Gaussian radius of curvature. This is the geometric mean $\sqrt{h_1 h_2}$ of the two principal radii of curvature h_1 and h_2. This number, which depends only on k, will be denoted by h(k). The notation will include a sign which is + for a point where the two radii have the same sign and the surface is convex toward the observation point, and is j for a point when the two radii are of opposite sign. The contribution of a stationary point k to the integral is, after simplification

$$\frac{-jh}{4\pi r} \quad H_k H_k^{\dagger} \; M \; e^{-jk \cdot r} \tag{15}$$

It is interesting to remark that $H_k^{\dagger}M$ may be interpreted as the reaction of the source ($J_e(r)$, $J_m(r)$) and the field of the characteristic wave $\check{F}_{-k}(r)$ in the transpose medium (H_0 reversed). Furthermore, using the complete characteristic field (9) we can write the total resulting field

$$\boxed{F(r) = \Sigma \frac{-jh}{4\pi r} \quad (H_k^{\dagger}M) \; F_k(r)} \tag{16}$$

where the vector k runs over the stationary points on Σ_+ and **h** is a function of k. This is the final formula. It is interesting to compare it with a similar formula for the far field of an antenna in free space

$$F(r) = \frac{-jk}{4\pi r} \left[J \cdot u \; F_{k,u}(r) + J \cdot v \; F_{k,v}(r) \right] \tag{17}$$

Here u and v are two vectors of length $\sqrt{\eta_o}$ normal to k and at right angle to each other. The vector k has the direction of r. The characteristic plane waves $F_{k,u}$ and $F_{k,v}$ have their **E** vectors at the origin respectively equal to u and v and as a result their Poynting vectors are equal to one. The scalar products J.u and J.v are the reactions of the source, here represented by an electric current density J, with the fields $F_{-k,u}$ and $F_{-k,v}$.

The essential differences between (17) and (16) are

1. In (17) any polarization is compatible with a vector k while in (16) the characteristic polarization is a function of k.
2. In (17) the direction k coincides with that of r while in (16) it does not.
3. In (16) a variable number of characteristic waves (up to 4) contribute to the far field. They have different phase velocities and produce an interference pattern in the radial direction.
4. The wave number k in (17) is replaced in (16) by the Gaussian radius of curvature h(k) at point k on Σ.

The function h(k) represents the gross features of the radiation pattern (neglecting the source directivity and polarization). It can be derived from Σ by the construction shown in Figure 2: the normal to the surface Σ at point k cuts the axis at point I, the parallel through I to the tangent at point k cuts the vector k at point L. If we call k' the distance from L to point k the Gaussian radius of curvature is simply

$$h^2 = k\, k' \frac{d\theta}{d\alpha} \tag{18}$$

In Figure 3 the function $h(\theta)$ has been sketched for one sheet of a dispersion surface represented in Figure 2. Figure 3 also shows the variation of α (ray direction) versus θ (wave-vector direction). For a given α there are sometimes 3 values of θ. The slope of the curve α vs θ and the dispersion surface can be used for a slide rule evaluation of h by means of formula (18).

The radius h becomes infinite for the inflexion points $k(\theta_1)$ and $k(\theta_2)$ and the saddle point integration has to be refined for these particular points as was done by Arbel and Felsen[4].

The polarization and directivity of the source are both contained in the factor $H_k^\dagger M$ which depends essentially on the values of the function M(k) on the dispersion surface Σ. One could say that it depends on the manner in which the antenna in free space would "illuminate" the dispersion surface. The computation of $H_k^\dagger M$ is simplified by use of the Poincaré sphere representation of the polarization of H and M.

As a conclusion formulas (16) and (18) make it possible to sketch radiation patterns of arbitrary antennas in a magneto-plasma without recourse to high-speed computers.

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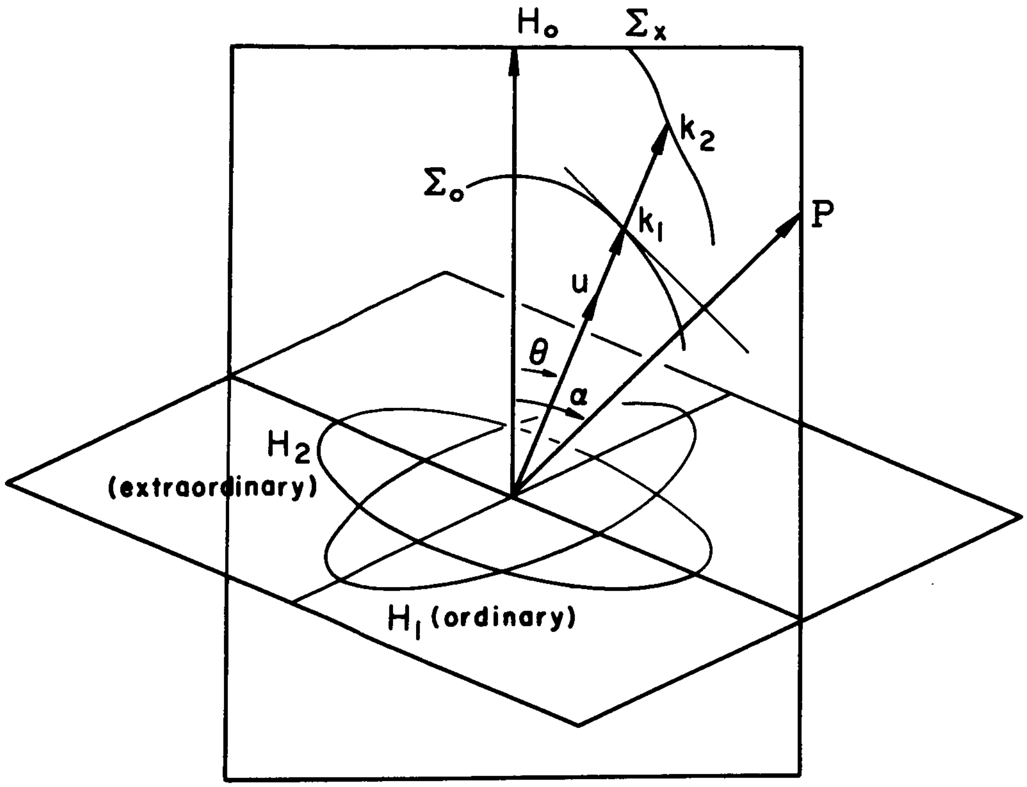

Figure 1. Polarization of the H-vector and relation of the average Poynting vector P to the wave vector k.

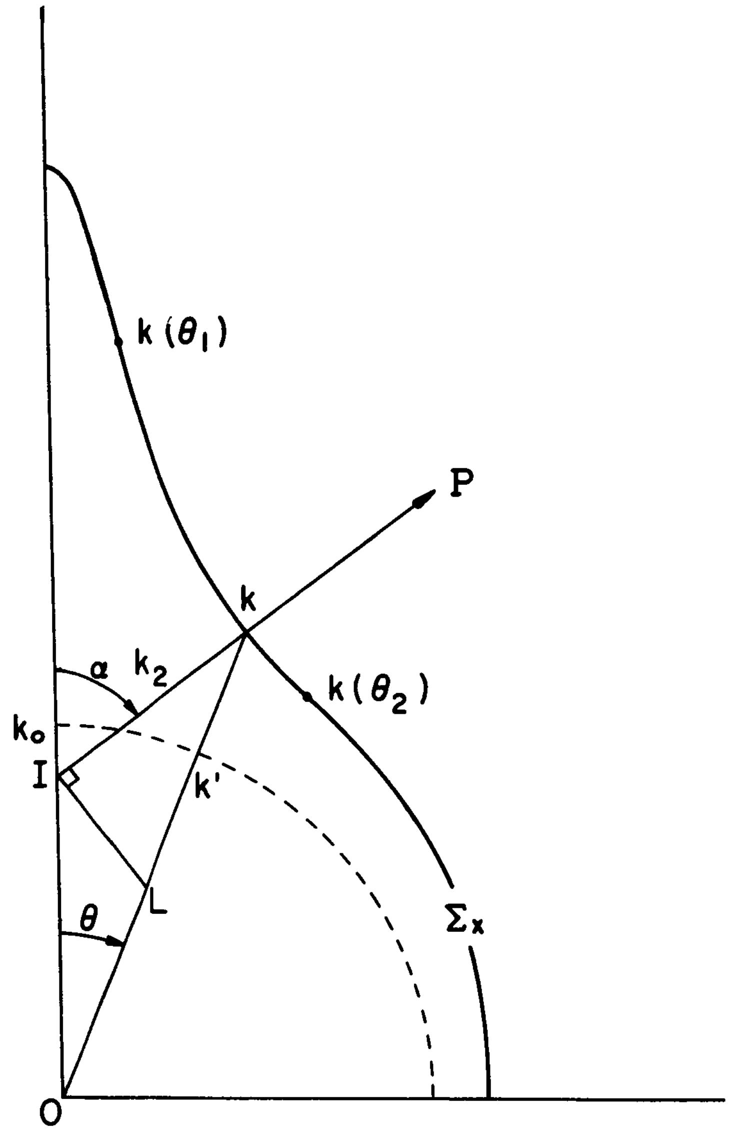

Figure 2. Examples of dispersion surface $K(\theta)$. (Extraordinary sheet Σ_x for $Y>1$, $X \lesssim 1$)

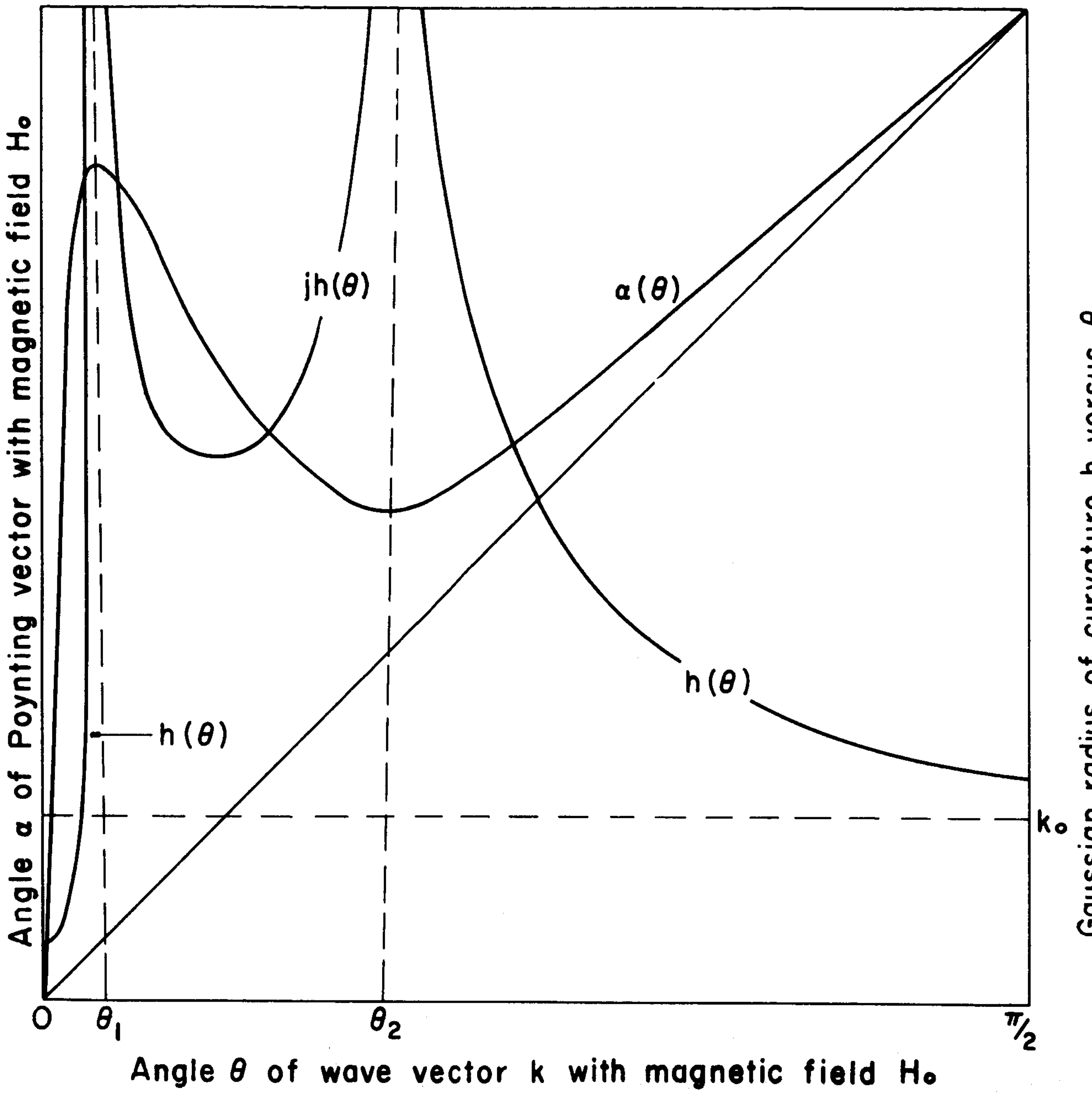

Figure 3. Sketch of the curves $\alpha(\theta)$ and $h(\theta)$ for the dispersion surface in Figure 2.

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